Session 1
Review of Newton's Laws of Motion

1.0 Importance of Physics

Testing airplanes requires pilots to know a lot more than just how to fly the plane; they must also know why an airplane flies. The science of flight is totally dependent upon physics. In fact, without a good understanding of physics, Orville and Wilbur Wright would never have gotten their Flyer off the ground.

Wright Flyer

Many "would-be" aircraft designers never took the time to study Newton's Laws, and as a result, built contraptions that flew worse than they looked.

"Would-Be" Design

Newton's Laws of Physics are still applied by aircraft designers every day for every type of aircraft. Using these laws, designers are able to determine such things as the overall shape of the aircraft, how many engines are required, how far it can go, and how much runway is needed to takeoff and land. All these areas must be addressed for the design to be successful.

NOTE:

The following types of aircraft are shown in the first video:

SAAB Draken
(supersonic fighter)

Aeromacchi Impala
(jet trainer)

Sikorsky S-55
(transport helicopter)

In order to make effective use of Newton's Laws, a brief review of each is in order.

Warning:

The assumption is made that the students have already been taught the development of Newton's Laws. The following presentations are meant to serve only as a refresher. Sections 5.1 through 5.4 of the accompanying text should be reviewed prior to starting the video.

Session 1
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****START VIDEO****

2.0 Newton's First Law

The First Law of Motion is often referred to as the Law of Inertia. The formal definition states:

> **"A body in motion at a uniform speed will remain in motion at that speed unless acted upon by an external force, and a body at rest will remain at rest unless acted upon by an external force."**

NOTE:

See the Operational Supplement at the end of this session for a description of speed and velocity.

A body in motion is exactly what seatbelts are designed to restrain. Seatbelts are known to save lives by preventing the vehicle occupant from continuing forward when the vehicle stops suddenly. The tendency for the occupant to continue forward is a classic case of inertia at work. The "external force" which acts upon the body comes in the form of the seatbelt.

Seatbelts at Work

For the speeds experienced in a car, the seatbelts/shoulder strap combination should provide sufficient stopping force for the occupant. However, since aircraft travel at much faster speeds and are free to move in three dimensions, a "five point" harness is often used; 2 shoulder straps, a right and left seatbelt, and a "negative-g" strap between the legs. The effectiveness of this arrangement can be seen on the sled track occupant.

Sled Track Test Subject

Each belt must be made of the proper material and to the correct size to provide enough external force to limit the pilot's movement. Determining the size of forces is the topic of Newton's Second Law.

NOTE:

The rapid deceleration rate caused Col. Stapp's eyes to hemorrhage, giving him two completely red eyes.

3.0 Newton's Second Law

Newton's Second Law of Motion relates force to acceleration. The formal definition is:

> **"Force is equal to mass times acceleration, or $F = ma$."**

NOTE:

See the Operational Supplement at the end of this session for discussion of acceleration.

An everyday example of this law occurs when we step on the scale to weigh ourselves.

Second Law at Work

The force can be measured directly as our weight, refered to as F_w. Additionally, on Earth, the acceleration of gravity is found to be 32.2 ft/sec^2. Substitution into the

$$F = ma,$$

equation and rearranging where $a = g$

$$m = \frac{F_w}{g}$$

this equation yields our mass.

NOTE:

The development of the universal gravitational constant is contained in the Operational Supplement.

The significance of knowing our mass comes to light when we are not subjected to the Earth's gravity. A man standing on the Earth has the same mass as he would standing on the moon. However on the moon he would weigh 1/6th of what he would weigh on the Earth. The difference in his weight comes from the differences in the gravitational acceleration constants between the Earth and the moon. The moon's gravitational acceleration is only 1/6th that of the Earth's. As a result, his weight is only 1/6th of his Earthly weight. In aviation, the Earth's gravitational acceleration is referred to as a "g." Often times a pilot may feel the effect of more (or less) than 1 "g."

As an aircraft maneuvers, the pilot experiences a change in the "g-factor." This factor is multiplied times the standard gravitational acceleration of 32.2 ft/sec^2. A mathematical expression of this would look like

$$F = ma$$

$$F_w = mg \ \ (\text{"}g\text{" factor})$$

Caution:

The "g" factor is actually a result of centripetal acceleration. The equation for this is

$$F = \frac{mV^2}{R}$$

However, for this application the "g" factor can be envisioned as simply a multiplication factor.

Example: The pilot in the video weighs 155 pounds. To determine his mass,

NOTE:

Descriptions of this and other acceleration factors are contained in the Operational Supplement.

$$F_w = mg$$

$$155 \text{ lbs} = m\ (32.2 \text{ ft/sec}^2)$$

therefore

$$m = \frac{F_w}{g} = 155 \text{ lbs}/32.2 \text{ ft/sec}^2$$

or

$$m = 4.8 \text{ slugs}$$

****STOP VIDEO after pilot talks about slugs****

NOTE:

The use of slugs as a unit of measurement may be foreign to some students. See page 4.2 of the text for a complete definition of a slug and its equivalent in the metric system.

During a "2-g" turn, the pilot's weight can be found by:

$$F_w = mg(\text{"}g\text{" factor})$$

$$F_w = (4.8 \text{ slugs})\ (32.2 \text{ feet/sec}^2)\ (2)$$

F_w = 309 pounds

NOTE:

The mass actually comes out to 4.8137 slugs. Due to round-off error, the 2*g* turn results in 309 lbs instead of the more exact 310 lbs.

****START VIDEO****

From this example, it can be seen that a "*g*" factor is purely a multiplication factor used to determine an increase in weight. Since this increase in weight acts towards the pilot's feet, the force may cause the blood to leave his upper body causing him to black out. As a result, he wears an "anti-*g*" suit to provide an opposing force on his legs keeping the blood in his head and chest. Opposing forces is the subject of Newton' third law.

4.0 Newton's Third Law

The third law of motion is often thought of as the law of action and reaction. Specifically it states

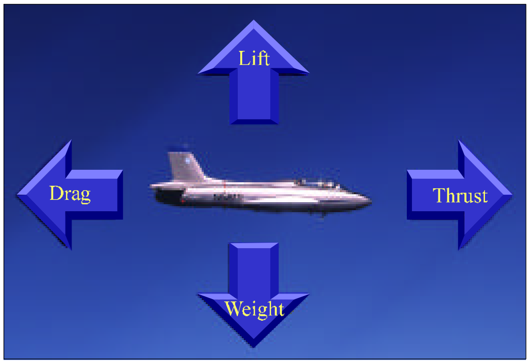
> **"When one object exerts a force on another, the second object must exert an equal and opposite force on the first."**

The simplest example of this is when we stand from sitting in a chair. We place our feet on the floor, use our legs to push against the floor, and push ourselves up. If the earth, or floor, didn't push back with an equal amount of force, we would fall into the earth (earth pushing back with less force) or we would be propelled into the air (earth pushing back with more force). The same principle applies to a jet engine. Thrust is the force produced by the hot gas coming out the back of the engine. Since Newton's third law must also be obeyed here, the air exerts a force equal to the thrust but in the opposite direction, propelling the jet forward in the same manner as the floor exerting a force on your legs allows you to stand.

Thrust is just one of four primary forces which act upon an aircraft in flight. The plane can't violate the third law, therefore thrust must be opposed by an equal and opposite force. This second force is called drag. Drag is the resistance of the atmosphere to the aircraft, like you feel when you put your hand out the window of a moving car. When drag and thrust are equal, the aircraft is no longer accelerating, but remains at the same speed since these forces are equal. If thrust is increased by adding more power, the aircraft will initially accelerate to a new speed. However as the plane's speed increases, so does the drag and eventually, thrust and drag will again be equal, but at a faster speed.

The remaining two forces on the aircraft highlight how a plane stays in the air. Lift is the force provided by the wings as the plane moves forward through the air. If lift is the force which causes a plane to rise, then it seems logical the opposing force would act in a downward direction. Not surprisingly, this force is the plane's weight. Now it may seem strange, but lift is always equal to weight, otherwise the aircraft couldn't stay in the air.

The aircraft is controlled by changing the lift forces over the wings and tail. Moving the control stick forward or backward causes more or less lift on the tail causing the nose to move up or down. Likewise moving the stick from side to side causes more lift on one wing, which results in a roll.



Forces in Flight

All four of these forces are actually dependent upon each other and in a future session, their interrelationships will be explored.

5.0 Summary

Newton's three laws were highlighted here to provide the student with an exposure into only a few areas where the laws come into play in aviation. The pilot's restraint system reviewed the principle of inertia, the "g" factors emphasized the change in force with a change in acceleration, while the jet engine and the forces of flight showed the concept of action and reaction. All of these laws can be demonstrated in the following example.

6.0 Measures of Performance

1 What does Newton's first law state?

ANSWER

"A body in motion at a uniform speed will remain in motion at that speed unless acted upon by an external force, and a body at rest will remain at rest unless acted upon by an external force."

2 What does Newton's second law state?

ANSWER

"Force is equal to mass times acceleration, or $F = ma$."

3 What does Newton's third law state?

ANSWER

"When one object exerts a force on another, the second object must exert an equal and opposite force on the first."

4 What is a "*g*"?

ANSWER

In aviation, the earth's gravational acceleration is referred to as a "*g*".

5 What is a "*g*" factor?

ANSWER

The "*g*" factor is actually a result of centripetal acceleration.

7.0 Example

Problem:

The pilot in the video said he weighs 155 pounds. The restraint system in the aircraft consists of five seatbelts (two shoulder belts, two lap belts and one negative "*g*" belt). How much force does each belt have to withstand to keep him from hitting the instrument panel if he experiences a positive "*g*" factor of +12 when the plane comes to a rapid stop during a crash landing?

Solution:

1. The pilot's mass is found by use of the second law:

$$F = ma$$

On Earth, one "*g*" is the acceleration of gravity (32.2 ft/sec^2) and the force is equal to his weight. Therefore, this mass is

$$w = mg$$

$$155 \text{ pounds} = m\,(32.2 \text{ feet/sec}^2)$$

$$m = 4.8 \text{ slugs}$$

2. Again using the second law, at a "*g*" factor of +12, the force is now

$$w = mg\,(\text{"}g\text{" factor})$$

$$w = (4.8 \text{ slugs})(32.2 \text{ feet/sec}^2)(12)$$

$$w = 1855.7 \text{ pounds}$$

3. This total force can be divided among the seat belts.

Caution:

One of the seatbelts is a "negative g" belt. Since the question states a "positive g-factor of 12" this belt should not be included in the calculations.

1855.7 pounds / 4 belts = 463.7 pounds per belt

4. Therefore each belt must be designed to be able to provide an opposing force of 463.7 pounds to contain the pilot's inertia.

NOTE:

The "negative *g"* belt only provides an anchor point for the lap belt and shoulder belt. This anchor prevents the belts form slackening during negative *g* maneuvers.

8.0 Suggested Activities

1 Have each student weigh themselves and determine their mass from the relationship

$$F_w = mg$$

$$m = \frac{F_w}{g}$$

2 Have each student determine how much they would weigh during a 2*g*, 4*g* and 9*g* turn.

Session 1
Review of Newton's Laws of Motion

Operational Supplement

<u>Speed and Velocity</u>

The simplest kind of motion that an object can have is a uniform motion in a straight line. This means an object moving in this manner is moving with a constant *velocity*. Constant velocity implies not only constant *speed*, but, unchanging *direction* as well. For this reason velocity is a vector quantity.

The speed of a moving body is the distance it moves per unit time in any arbitrary direction. If the speed is uniform, the object moves an equal distance in each successive unit of time. Speed is a scalar measurement since the direction of motion is immaterial. Whether or not the speed is constant, the *average speed* is the distance the body moves divided by the time required for the motion:

$$V_{avg} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t} \qquad (1)$$

where Δs is the distance traveled, V_{avg} is the average speed, and Δt is the elasped time. The British system unit of speed is the foot per second (ft/sec); the SI unit is the meter per second (m/sec); many other units are common, such as the mile per hour (mi/hr), centimeter per second (cm/sec), knot (kts), etc.

The terminology used above is very important. The concept of *speed* does not involve the idea of direction. A body moving with constant speed may move in a straight line or in a circle or in any one of an infinite variety of paths so long as the distance moved in any unit of time is the same as that moved in any other equal unit of time. The concept of *velocity* includes the idea of direction as well as magnitude. Hence we must consider the *displacement* of a body and not merely the *distance* traveled. The definition of average velocity, then, is given by:

$$\bar{V}_{avg} = \frac{\overline{s_2} - \overline{s_1}}{t_2 - t_1} = \frac{\Delta \bar{s}}{\Delta t} \qquad (2)$$

The defining equation for average velocity (Equation 2) is different from the equation for average speed (Equation 1) in that $\bar{v}$ and $\bar{s}$ are vector quantities. The bar over the symbol is used to emphasize this fact. Constant velocity is a particular case of constant speed. Not only does the distance traveled in unit time remain the same, but, the direction is unchanged as well.

<u>Accelerated Motion</u>

Objects seldom move with constant velocity. In almost all cases, the velocity of an object is continually changing in magnitude or in direction or both. Motion in which the velocity is changing is called *accelerated motion*, and the rate at which the velocity changes is called the *acceleration*. The velocity of a body may be changed by changing the speed, by changing the direction, or by changing both speed and direction. If the direction of the acceleration is parallel to the direction of motion, only the speed changes, while, if the acceleration is at right angles to the direction of motion, only the direction changes. Acceleration in any other direction produces changes in both speed and direction. For the present, we will confine our attention to the simplest type of accelerated motion, called *uniformly accelerated motion*. In this case the direction is always the same and only the speed changes at a constant rate in the direction of the original motion. The acceleration in this case is equal to the rate of change of speed, since there is no change in direction. The acceleration is *positive* if the speed is increasing, *negative* if the speed is decreasing. Negative acceleration is sometimes called *deceleration*.

The acceleration of a body is defined as the time rate of change of velocity. Using algebraic symbols to represent average acceleration, the defining equation is written:

$$a_{avg} = \frac{V_f - V_i}{t} = \frac{\Delta V}{\Delta t} \tag{3}$$

where a_{avg} represents the average acceleration, V_f the final velocity, V_i the initial velocity, and t the elapsed time. Since units of acceleration are obtained by dividing a unit of velocity by a unit of time, it may be seen that the British unit of acceleration is the foot per second per second (ft/sec^2) and the SI unit is the meter per second per second (m/sec^2).

<u>Uniformly Accelerated Motion</u>

Because they are often encountered, it is convenient to remember and list the equations for the special cases which apply to a body moving with constant acceleration in a straight line. If both sides of Equation 3 are multiplied by t, we obtain:

$$V_f - V_i = at \tag{4}$$

which expresses the fact that the change in speed is equal to the rate of change in speed multiplied by the time during which it is changing. The distance traveled during any time is gotten by multiplying Equation 1 by t:

$$s = V_{avg}t \tag{5}$$

But, the average speed V_{avg} must be obtained from the initial and final speeds V_i and V_f. Since the speed changes at a uniform rate, the average speed is equal to the average of the initial and final speeds:

$$V_{avg} = \frac{V_i + V_f}{2} \tag{6}$$

By combining these equations, two other useful equations can be obtained. Eliminating V_f and V_{avg}, we obtain:

$$s = V_i t + \tfrac{1}{2}at^2 \tag{7}$$

If we eliminate V_{avg} and t, we obtain:

$$V_f^2 = V_i^2 + 2as \tag{8}$$

Of these five equations, Equation 5 is true for all types of motion; the remaining four equations hold only for *uniformly accelerated linear motion.*

<u>Universal Gravitation</u>

In addition to the three laws of motion, Newton formulated a law of great importance in mechanics, the law of *universal gravitation. Every particle in the universe attracts every other particle with a force that is directly proportional to the product of the masses of the two particles and inversely proportional to the square of the distance between them.* This relation may be expressed symbolically by the equation:

$$F = \frac{Gm_1 m_2}{s^2} \tag{9}$$

where F is the force of attraction, m_1 and m_2 are the respective masses of the two particles, s is the distance between them, and G is a constant called the *gravitational constant.* The value of G depends on the system of units used in Equation 4. If the force is expressed in newtons, mass in kilograms, and distance in meters, G has the value $6.67 \times 10^{-11} m^3/kg\text{–sec}^2$. If the force is expressed in pounds, mass in slugs, and distance in feet, G has the value $3.42 \times 10^{-8} ft^4/lb\text{–sec}^4$.

Newton checked his law of gravitation by calculation and observation of the orbit of the moon. With the approximate data at his disposal, he still found reasonable agreement between his computations and his observations. Careful subsequent experimentation and measurement of the force of attraction between small bodies in the laboratory has further established the validity of the law of universal gravitation and led to the determination of the value of G given above.

Uniform Circular Motion

In *uniform circular motion,* the velocity vector remains constant in magnitude while the direction continually changes. Just as a force is required to change the speed of an object, so a force must also act to cause a change in the direction of the motion. Whenever the net force on a body acts in a direction other than the original direction of motion, it changes the direction of the motion. Such acceleration is very common, for it is present whenever a car turns a corner, an airplane changes its direction, or in any other similar motions.

Central Acceleration

When an object is moving in a circular path with constant speed, its *velocity* is continually changing. The acceleration produces a change in direction but no change in speed. Therefore, the acceleration must always be at right angles to the motion, since any component in the direction of the motion would produce a change in speed. The acceleration is always directed toward the center of the circle in which the body moves. It is constant in magnitude but contiually changing direction. In Figure 1 a body is moving with uniform speed, v, and constant angular speed, ω, in a circular path. The linear speed and angular speed are related by the equation:

$$V = \omega r \tag{10}$$

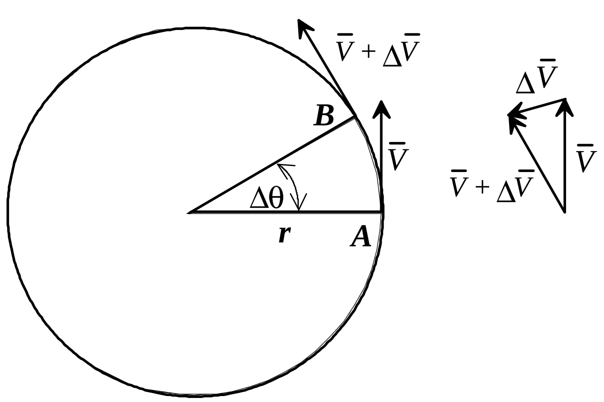


Figure 1 Uniform Circular Motion

where r is the radius of the circular path. The velocities of the object at points A and B are, respectively, $\bar{V}$ and $\bar{V} + \Delta\bar{V}$, equal in magnitude, but, differing in direction by a small angle $\Delta\theta$. In the vector triangle, $\Delta\bar{V}$

represents the change in velocity in the time Δt required for the object to to move from A to B. If the angle $\Delta\theta$ is small, the chord is approximately equal to the arc, and thus:

$$\Delta V = V\Delta\theta \tag{11}$$

But, since:

$$\Delta\theta = \omega\Delta t \tag{12}$$

Hence:

$$\Delta V = V\,\omega\,\Delta t = \omega^2 r\,\Delta t = \frac{V^2}{r}\,\Delta t \tag{13}$$

and:

$$\frac{\Delta V}{\Delta t} = \omega^2 r = \frac{V^2}{r} \tag{14}$$

As Δt is made smaller, the approximation in this equation becomes less and less, and the direction of $\Delta\bar{V}$ becomes more nearly perpendicular to that of $\bar{V}$. As Δt approaches zero, the instantaneous acceleration is found to be directed toward the center of the circle and is given by:

$$a_c = \frac{dV}{dt} = \omega^2 r = \frac{V^2}{r} \tag{15}$$

This equation states that the acceleration increases as the speed is increased and, for a given speed, is greater for a shorter radius. The acceleration is at right angles to the velocity and hence is directed toward the center of the circle. If the angular speed in Equation 15 is expressed in radians per second, then the units of a_c then depend upon the units in which r and V are expressed in. If the units of r are in feet and V in feet per second, then the units of a_c are in ft/sec^2 . If the units of r are in meters and V in meters per second, then the units of a_c are in m/sec^2.

<u>Centripetal Force</u>

According to Newton's laws of motion, any object that experiences an acceleration is acted upon by an unbalanced force, a force that is proportional to the acceleration and in the direction of the acceleration. The net force that produces the central acceleration is called the *centripetal force* and is directed toward the center of the circular path. Every body that moves in a circular path does so under the action of a centripetal force. A body moving with uniform speed in a circle is not in equilibrium. From Newton's second law, the magnitude of the centripetal force is given by:

$$F_c = ma_c = m\frac{V^2}{r} = m\,\omega^2 r \tag{16}$$

where m is the mass of the moving object, V is its linear speed, r is the radius of the circular path, and ω is the angular speed. If m is in slugs, V in ft/sec, and r in ft, then F_c is in lb. If m is in m/sec, V in m/sec, and r in m, then F_c is in newtons.

An inspection of Equation 16 discloses that the centripetal force necessary to keep a body in a circular path, as shown in Figure 2, is directly proportional to the square of the speed at which the body moves and inversely proportional to the radius of the circular path. If the speed is doubled, keeping the radius constant, the centripetal force becomes four times as great. If instead, the radius is cut in half, with the speed remaining constant, the centripetal force increases to twice as great. If at any instant the cord in

Figure 2 breaks, eliminating the centripetal force, the rock will retain the velocity it has at the instant the cord breaks and travel at constant speed along a line tangent to the circular path at that point. The act of throwing a baseball follows the exact same principle.

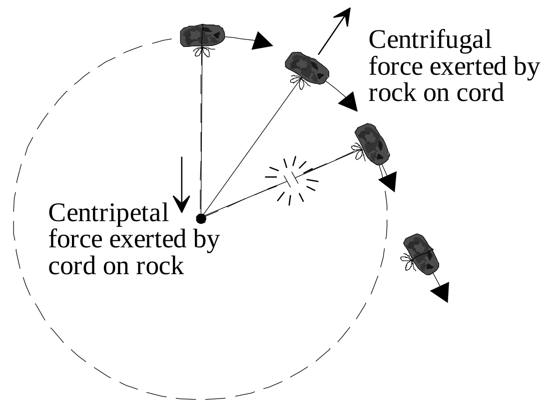


Figure 2 Centripetal and Centrifugal Forces

Work has been defined as the product of a force and a displacement in the direction of the force. Since centripetal force acts at right angles to the direction of motion, there is no displacement in the direction of the centripetal force, and it accomplishes no work. No energy is expended on or by an object while it is moving at constant speed in a horizontal circular path. This conclusion is consistent with the observation that, if the speed is constant, the kinetic energy of the body is also constant.

Centrifugal Force

Newton's third law expresses the fact that for every force that is exerted on a body, there is a second force, equal in magnitude but opposite in direction, acting on a second body. The cord that constrains the rock to a circular path in Figure 8.2 exerts the centripetal force on the rock that changes its velocity. In reaction against this change of motion, the rock pulls outward on the cord with a force called the *centrifugal force.*

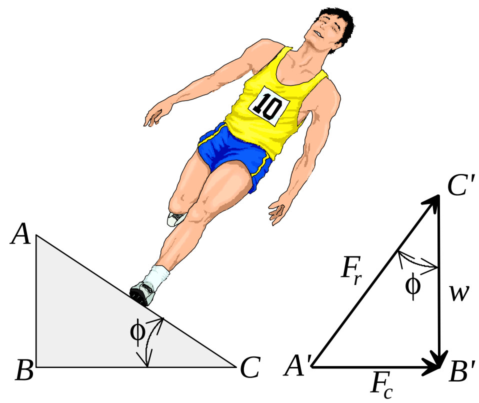
As the speed of a flywheel increases, the force needed to hold the parts of the wheel in circular motion increases with the square of the angular speed, as indicated by Equation 8.7. If the speed becomes high enough, the cohesive forces between the molecules of the material that the flywheel is made of are no longer sufficient and the wheel disintegrates, the parts flying off along tangent lines like mud from an automobile tire. Whenever news reports of an aircraft engine failure during flight, it is often due to rotating fan blades in the engine coming apart from the stresses created by the combination of heat and rotational forces.

When a container of liquid is being whirled in a horizontal circular motion, the container exerts an inward force on the liquid sufficient to keep it from spilling out. The bottom of the container presses on the layer of liquid next to it; that layer in turn exerts a force on the next; and so on. In each layer, the pressure must be the same all over the layer or the liquid will not remain in the layer. If the liquid is of uniform density, each element of volume with a mass m in a given layer will experience an inward force (mV^2/r) just great enough to maintain it in that layer and there will be no motion of the liquid from one layer to another. If, however, the layer is made up of a mixture of particles of different densities, the force required to maintain a given element of volume in that layer will depend upon the density of liquid in that element. Since the inward force is the same on all the elements in a single layer, there will be motion between the layers. For those elements which are less dense than the average, the central force is greater than that necessary to hold them in the layer; hence they are forced inward. For the elements more dense than the average, the central force is insufficient to hold them in the layer and they will move to a layer farther out. As rotation continues, the elements of the mixture become separated, with the least dense nearest the axis of rotation and the most dense farthest from the axis. This behavior is utilized to our advantage in the *centrifuge*, a device for separating liquids of different densities. Very high speed centrifuges may be used to separate gases of different densities.

Airplane pilots, who put their aircraft into a very tight turn or pull out of a steep dive at high speed, often experience centripetal accelerations several times as large as the acceleration due to gravity. Under these circumstances, the flow of blood to the pilot's brain is decreased unless other measures are taken to counteract these forces. Without a "*g*-suit" strapped to his torso, these high g-forces can cause the pilot to lose consiousness ("black out") during such periods of maximum acceleration.

Turns

A runner, in going around a curve, leans inward to obtain the centripetal force that causes him to turn as shown in Figure 3. The track must exert an upward force sufficient to sustain his weight, while at the same time it must provide a horizontal centripetal force. If the track is flat, the horizontal force must be entirely frictional. In that case, the frictional force may not be large enough to enable a sharp turn if the surface of the track were smooth. If the track is tilted from the horizontal, a portion of the horizontal force can be sustained by the horizontal component of the reaction force provided by the track surface while the remainder is still supplied by friction. If the angle of banking is properly selected, the force the track exerts, which is perpendicular to its surface, will be sufficient to provide the necessary horizontal force without friction.



(a) (b)

Figure 3 A Banked Turn

For this ideal case, as shown in Figure 3, the reaction force F_r of the track is perpendicular to the surface AC. The force due to the weight of the runner w is directed vertically downward. The resultant force F_c is the horizontal centripetal force. In the force triangle in Figure 3, the angle ϕ is the angle of bank of the track:

$$\tan v = F_c/w = \frac{mV^2/r}{mg} = \frac{V^2}{rg} \tag{17}$$

Equation 17 indicates that, since the angle of bank depends upon the speed, the curve can be ideally banked for only one speed. At any other speed, the force of friction must be depended upon to prevent slipping.

Let us now consider the turning flight of an airplane. In particular, we will only examine three specialized cases: (1) a level turn, (2) a pullup, and (3) an inverted pulldown (split-s). A study of the generalized motion of an airplane along a three-dimensional flight path is beyond the scope of this series.

A level turn is illustrated in Figure 4. Here the wings of the airplane are banked through the angle ϕ; hence the lift vector is inclined at the angle ϕ to the vertical. The bank angle ϕ and the lift L are such that the component of lift in the vertical direction exactly equals the force due to weight of the aircraft:

$$w = L\cos\phi \tag{18}$$

and therefore the airplane maintains a constant altitude, moving in the horizontal plane. The resultant of L and F_w leads to a resultant centripetal force F_c which acts in the horizontal plane causing the airplane to turn in a circular path with a radius of curvature equal to R and a turn rate of ω.

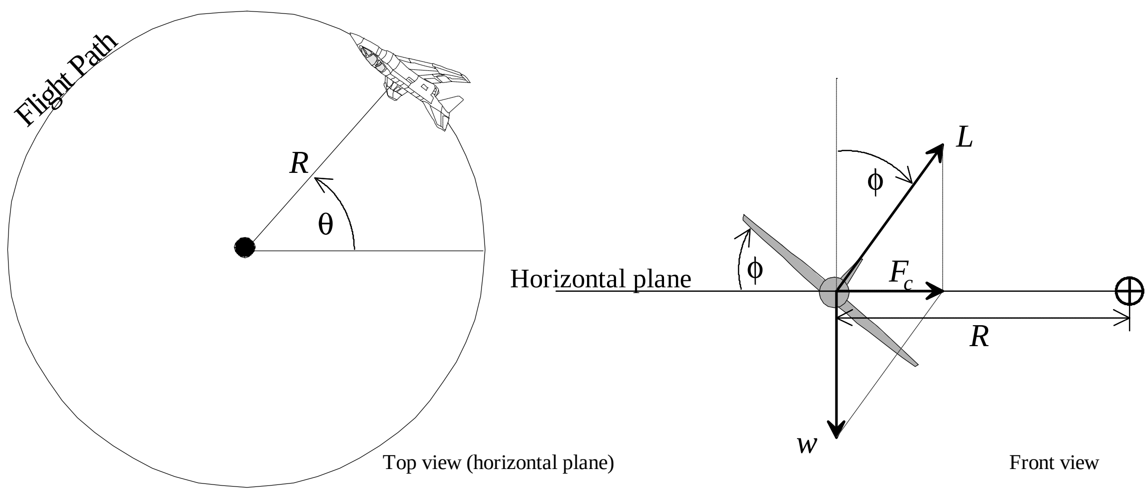


Figure 4 An Airplane in a Level Turn

From the force diagram in Figure 4, the magnitude of the resultant force is:

$$F_c = \sqrt{L^2 - w^2}$$

If we introduce a new term, the *load factor n*, defined as:

$$n \equiv L/w$$

and combine the above equation with Equation 18, we can show that load factor can be expressed as a function of bank angle only:

$$n = \frac{L}{L\cos v} = 1/\cos v \tag{19}$$

Load factor is usually quoted in terms of "*g*'s"; for example, an airplane with lift equal to five times the weight is said to be experiencing a load factor of 5 *g*'s. Hence, the centripetal force can be written as:

$$F_c = w\sqrt{n^2 - 1} \tag{20}$$

The airplane is moving in a circular path at the velocity V; therefore, the centripetal force can also be expressed from Equation 16 as:

$$F_c = m\frac{V^2}{R} = \frac{wV^2}{gR} \tag{21}$$

Combining Equations 20 and 21 and solving for R, we have:

$$R = \frac{V^2}{g}\sqrt{n^2 - 1} \tag{22}$$

And, the turn rate ω = V/R. Thus, from Equation 21, we have:

$$z = \frac{g}{V}\sqrt{n^2 - 1} \tag{23}$$

For the maneuvering performance of an aircraft, both military and civilian, it is frequently advantageous to have the smallest possible R and the largest posssible ω. Equations 22 and 23 show that, to obtain both a small turn radius and a large turn rate, we must have:

1. The highest possible load factor ($n = L/w$)
2. The lowest possible velocity

Consider the second case of a pullup maneuver where the airplane, initially in straight and level flight, suddenly experiences an increase in lift. Since the lift is greater than the weight of the airplane in this case, the airplane will begin to accelerate upward in a "vertical turn" or circular path in the vertical plane as shown in Figure 5. From the force diagram in Figure 5, the centripetal force F_c is vertical and is given by:

$$F_c = L - w = w(n - 1) \tag{24}$$

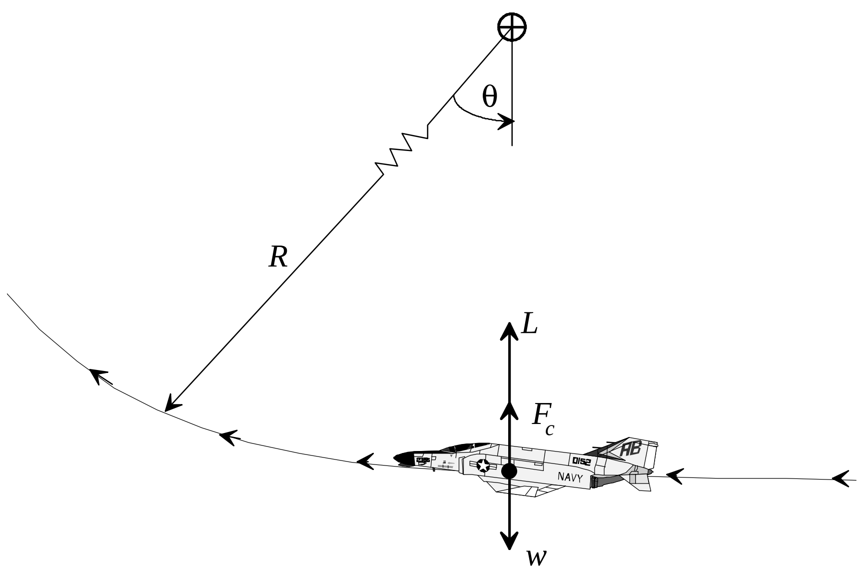


Figure 5 The Pullup Maneuver

We have from Equation 21:

$$F_c = m\frac{V^2}{R} = \frac{wV^2}{gR} \tag{21}$$

Combining Equations 21 and 24 and solving for R we get:

$$R = V^2/g(n-1) \tag{25}$$

And, the turn rate $\omega = V/R$. Thus, from Equation 25, we have:

$$\omega = g(n-1)/V \tag{26}$$

A related case is case 3, the inverted pulldown maneuver, illustrated in Figure 6. Here, an airplane, initially in straight and level flight, suddenly rolls to an inverted position, such that both L and F_w are pointing downward. The airplane will begin to turn, in the vertical plane, downward in a circular flight path with turn radius R and turn rate ω. By an analysis similar to the pullup above, the following results are easily obtained:

$$F_c = L + w = w(n+1) = \frac{wV^2}{gR} \tag{27}$$

$$R = V^2/g(n+1) \tag{28}$$

$$\omega = g(n+1)/V \tag{29}$$

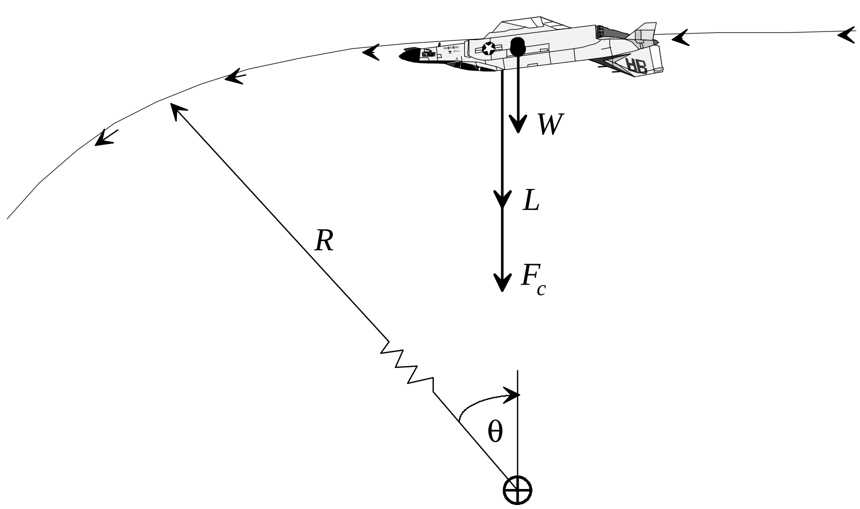


Figure 6 The Inverted Pulldown Maneuver

Considerations of turn radius and turn rate are particularly important to military fighter aircraft; everything else being equal, those airplanes with the smallest R and the largest ω will have definite advantages in air combat. High performance fighter aircraft are designed to operate at high load factors, typically from 5 to 9 g's; and if the turn is accomplished at the exact speed where the aerodynamic lift generated by the wing is sufficient to produce the maximum g at the minimum speed, the tightest turn will result with the aircraft possessing its highest energy level. This speed is often referred to as the "corner velocity" of the aircraft.

Curvilinear Motion

Frequently the net force acting on a body is neither parallel to the direction of its motion nor at right angles to that direction. In this case, neither the speed nor the direction remains constant. Such motion may be readily studied by considering two components of the acceleration, one parallel to the original direction of motion, the other perpendicular to that direction.

One of the most common of such motions is planetary motion, in which the force on the moving body is inversely proportional to the square of the radius and always directed toward a fixed point. The body travels in an ellipse, the fixed point being at one focus. The speed of the moving body is greatest when the body is nearest the focus, less when it is further away. This motion is called planetary motion because the planets move in this manner in their journeys around the sun. Comets have much more elongated elliptical paths that carry them outside the solar system at their furthest distance from our sun. Since electrified particles show a similar law of attraction, we should expect them to behave in the same manner as those moving under the action of gravitational forces.

Another simpler example of curvilinear motion that is closer to home is projectile motion. The science of the motion of projectiles is called *ballistics*. The simplest type of ballistic motion is that in which the projectile is given an initial velocity and then allowed to move under the influence of gravity alone. True projectile motion is that in which an object is given an initial velocity and then allowed to proceed under the action of gravity and also air resistance. Other objects which are self-propelled, such as rockets and missiles, move in the same manner as projectiles except that they do not depend upon an initial impulse alone, but also upon a sustained force throughout most of its flightpath. The initial speed of the rocket or missile may be quite low since it is continually gaining speed along its path.

All of these examples of curvilinear motion are outside the scope of this series. They are mentioned here to provide a knowledge of their existence.

Summary

In uniform circular motion: (a) the speed V is constant; (b) the direction of the motion is continually and uniformly changing; and (c) the acceleration a_c constant in magnitude and is directed toward the center of the circular path. The magnitude of the *central acceleration* is given by:

$$a_c = \frac{V^2}{r} = \mathrm{z}^2 r \tag{15}$$

where V is the linear speed, r is the radius, and ω is the angular speed.

The *centripetal force*, the inward force that causes the central acceleration, is given by:

$$F_c = m\frac{V^2}{r} = m\,\mathrm{z}^2 r \tag{16}$$

The *centrifugal force* is the outward reaction force exerted by the moving body on the agent of its centripetal force. The magnitude of the centrifugal force is equal to that of the centripetal force and opposite in direction.

The proper banking of a curve to eliminate the necessity for a horizontal frictional force is given by the relation:

$$\tan \mathrm{h} = \frac{V^2}{gr} \tag{17}$$

The *load factor* being pulled by an airplane in *level turning flight* is defined as:

$$n = L/w = 1/\cos\phi \tag{19}$$

The turn radius is given by:

$$R = \frac{V^2}{g}\sqrt{n^2 - 1} \tag{22}$$

and the turn rate is:

$$z = g\sqrt{n^2 - 1}\,/V \tag{23}$$

The turn radius and turn rate for a pullup is given by:

$$R = V^2/g(n-1) \tag{25}$$

$$\omega = g(n-1)/V \tag{26}$$

And, the turn radius and turn rate for an inverted pulldown is given by:

$$R = V^2/g(n+1) \tag{28}$$

$$\omega = g(n+1)/V \tag{29}$$

Often in curvilinear motion, the accelerating force is neither parallel nor perpendicular to the direction of motion. In this case, the acceleration produces change in both speed and direction of motion.